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**Math 3360 - Foundations of Algebra I**  
**Assignment Sheet 2**  
**Due in-class Monday April 21st**

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1. Let  $H_1 = \{e, (12)(34), (13)(24), (14)(23)\}$  and  $H_2 = \{e, (12)(34)\}$ .
  - (a) Prove that  $H_1$  is a normal subgroup of  $S_4$ .
  - (b) Prove  $S_4/H_1$  is isomorphic to  $S_3$ .
  - (c) Prove that  $H_2$  is a normal subgroup of  $H_1$  but not of  $S_4$ .
  
2. Prove that the group of integers under addition is not isomorphic to the group of rational numbers under addition.
  
3. Prove that  $\mathbb{Z}_2 \times \mathbb{Z}_2$  is not isomorphic to  $\mathbb{Z}_4$ .
  
4. Let  $G$  be a group.
  - (a) If  $H < G$  with  $[G : H] = 2$ , prove  $H \triangleleft G$ .
  - (b) If  $N \triangleleft G$ , prove  $NH < G$ .
  - (c) If  $H < G$  and  $N \triangleleft G$ , prove  $H \cap N \triangleleft G$ .
  - (d) Suppose  $G$  is finite and  $H$  is the only subgroup of order  $|H|$ . Prove  $H \triangleleft G$ .